Salt Lake Community College
Math 1060 Final Exam - Fall Semester 2009

Name: Key

This exam has three parts. Please carefully read the directions for each part. All problems are of equal point value. Students are NOT allowed to use books or notes.

Part I – You must complete this portion of the test without using a calculator. After you have finished part I, your instructor will give you the remaining parts of the exam.

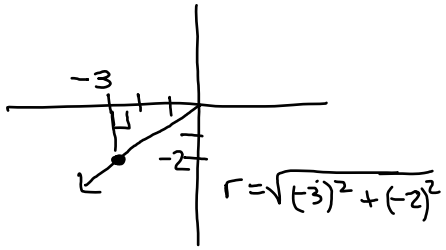
When simplifying answers, it is **not** necessary to rationalize denominators.

1) Find the exact values of all trigonometric functions for an angle α in standard position whose terminal side contains the point (-3,-2).

$\cos\alpha = -\frac{3}{\sqrt{13}}$ or $-\frac{3\sqrt{13}}{13}$ $\quad \sec\alpha = -\frac{\sqrt{13}}{3}$

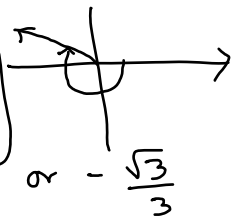
$\sin\alpha = -\frac{2}{\sqrt{13}}$ or $-\frac{2\sqrt{13}}{13}$ $\quad \csc\alpha = -\frac{\sqrt{13}}{2}$

$\tan\alpha = \frac{2}{3}$ $\quad \cot\alpha = \frac{3}{2}$



2) Find the exact value of each expression. If the expression is undefined , say so.

a) $\tan\left(-\frac{7\pi}{6}\right) = \frac{\sin\left(-\frac{7\pi}{6}\right)}{\cos\left(-\frac{7\pi}{6}\right)} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$



b) $\csc(2\pi)$ undefined (since $\sin 2\pi = 0$)

c) $\sec(45^\circ) = \frac{1}{\cos(45^\circ)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$ or $\sqrt{2}$

3) The position x of a weight attached to a spring is given by

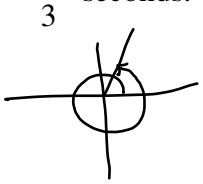
$x(t) = 4\sin(t) + 3\cos(t)$

where t is time in seconds. Find the exact position of the weight at time $t = \frac{7\pi}{3}$ seconds.

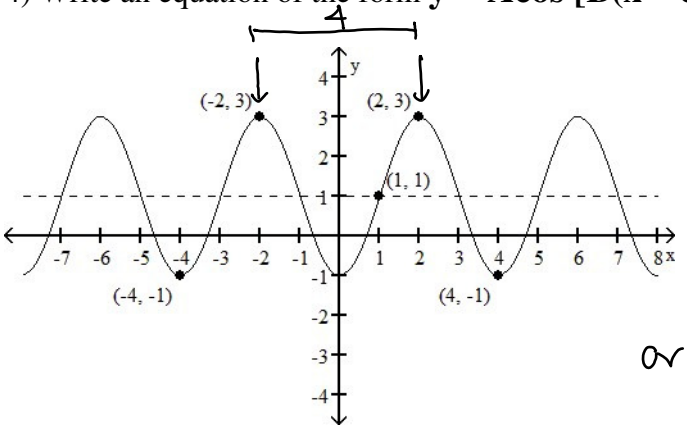
$x\left(\frac{7\pi}{3}\right) = 4\sin\left(\frac{7\pi}{3}\right) + 3\cos\left(\frac{7\pi}{3}\right)$

$= 4\left(\frac{\sqrt{3}}{2}\right) + 3\left(\frac{1}{2}\right)$

$= \boxed{2\sqrt{3} + \frac{3}{2}}$



4) Write an equation of the form **y = Acos [B(x – c)] + D** whose graph is give below.



Period = 4

$B = \frac{2\pi}{4} = \frac{\pi}{2}$

$y = -2\cos\left(\frac{\pi}{2}x\right) + 1$

or

$y = 2\cos\left[\frac{\pi}{2}(x+2)\right] + 1$

(other correct answers are possible)

5) Determine the amplitude, period, phase shift, and frequency for

$y = -3\sin\left(2x + \frac{\pi}{2}\right) - 1$

$y = -3\sin\left[2\left(x + \frac{\pi}{4}\right)\right] - 1$

Amplitude = 3

Period = $\frac{2\pi}{2} = \pi$

Period = π

Phase Shift = $-\pi/4$

Frequency = $\frac{1}{\pi}$

6) Use identities to simplify the following expression.

$$\frac{(\cos x \tan x + 1)(\sin x - 1)}{\cos^2 x}$$

$$= \frac{\left(\cos x \left(\frac{\sin x}{\cos x}\right) + 1\right)(\sin x - 1)}{\cos^2 x}$$

$$= \frac{(\sin x + 1)(\sin x - 1)}{\cos^2 x}$$

$$= \frac{\sin^2 x - 1}{\cos^2 x}$$

$$= -\frac{\cos^2 x}{\cos^2 x}$$

$$= -1$$

7) Prove that the following equation is an identity.

$$\tan^2(-x) - \frac{\sin(-x)}{\sin x} = \sec^2 x$$

$$\tan^2(-x) - \frac{\sin(-x)}{\sin x}$$

$$= (-\tan x)^2 + \frac{\sin x}{\sin x}$$

$$= \tan^2 x + 1$$

$$= \sec^2 x$$

8) Find the exact value of the following expression.

$\sin 40^\circ \cos 10^\circ - \cos 40^\circ \sin 10^\circ$

$= \sin(40^\circ - 10^\circ)$

$= \sin 30^\circ$

$= \boxed{\frac{1}{2}}$

9) Use an appropriate identity to find $\cos\alpha$.

$\cos 2\alpha = -\frac{1}{3}$ and $180^\circ < 2\alpha < 270^\circ \Rightarrow$ $90^\circ < \alpha < 135^\circ$, α in QII

$\cos\alpha = \boxed{-\sqrt{\frac{1}{3}}}$ or $-\frac{\sqrt{3}}{3}$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$-\frac{1}{3} = 2\cos^2\alpha - 1$

$\frac{2}{3} = 2\cos^2\alpha$

$\frac{1}{3} = \cos^2\alpha$

α in QII $\Rightarrow -\sqrt{\frac{1}{3}} = \cos\alpha$